

JPF2: Predicate Abstraction

CS 510/10

Predicate Abstraction

- **Extract a finite state model from an infinite state system**
- **Used to prove assertions or safety properties**
- **Successfully applied for verification of C programs**
 - **SLAM (used in windows device driver verification)**
 - **MAGIC, BLAST, F-Soft**

Example for Predicate Abstraction

```
int main() {  
    int i;  
  
    i=0;  
  
    while (even(i))  
        i++;  
}
```

C program

+

$p_1 \Leftrightarrow i=0$
 $p_2 \Leftrightarrow \text{even}(i)$

Predicates

=

```
void main() {  
    bool p1, p2;  
  
    p1=TRUE;  
    p2=TRUE;  
  
    while (p2)  
    {  
        p1=p1?FALSE:nondet();  
        p2=!p2;  
    }  
}
```

Boolean program

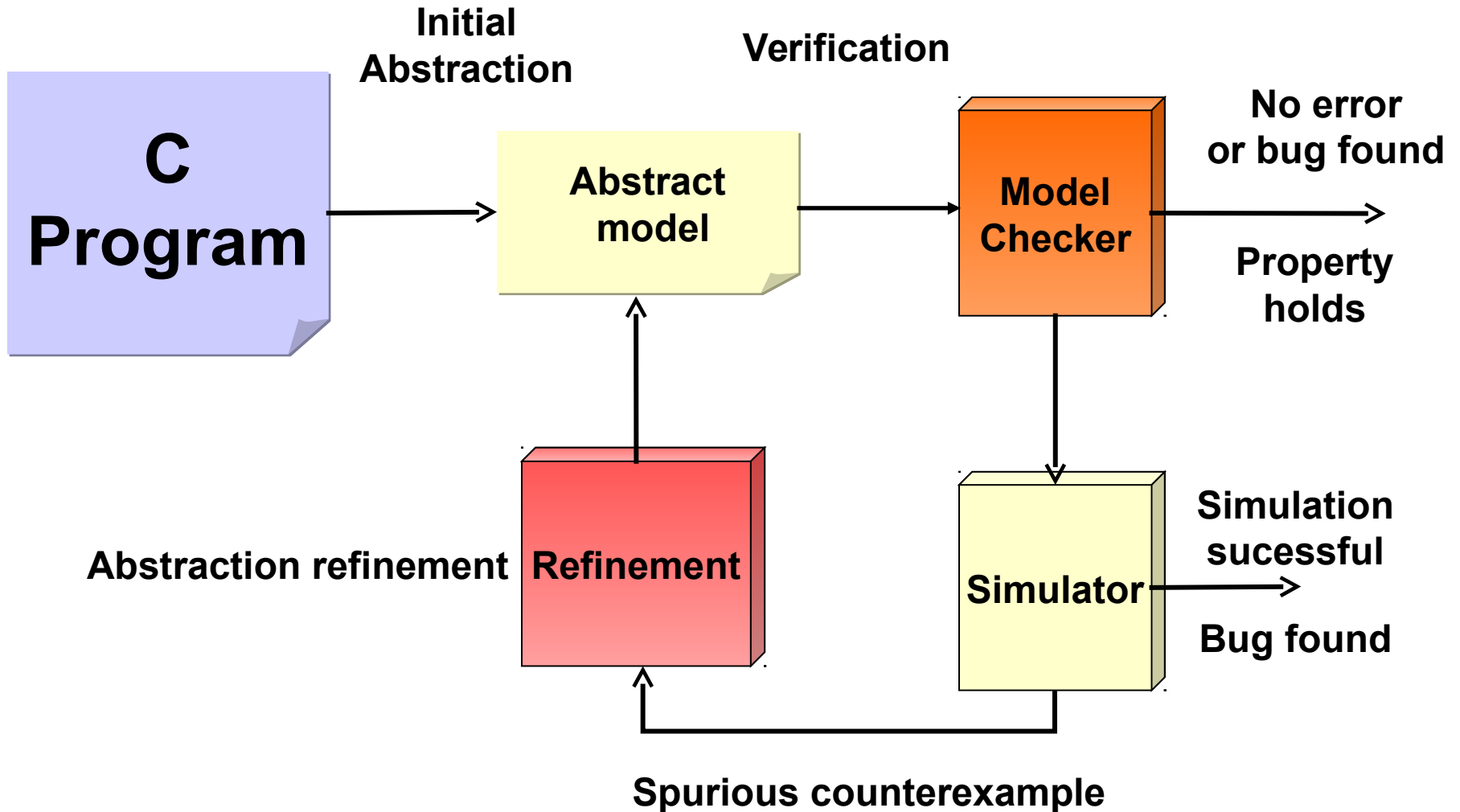
[Graf, Saidi '97]

[Ball, Rajamani '01]

Computing Predicate Abstraction

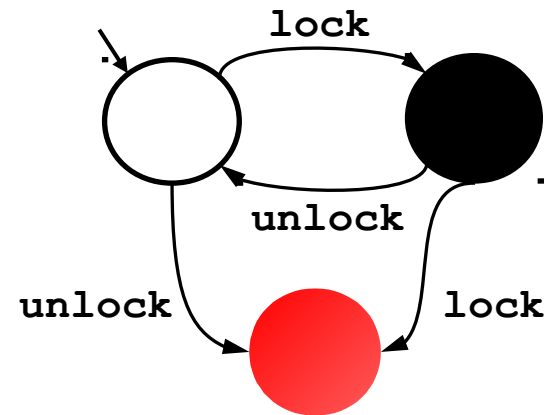
- **How to get predicates for checking a given property?**
- **How do we compute the abstraction?**
- **Predicate Abstraction is an over-approximation**
 - **How to refine coarse abstractions?**

Counterexample Guided Abstraction Refinement loop

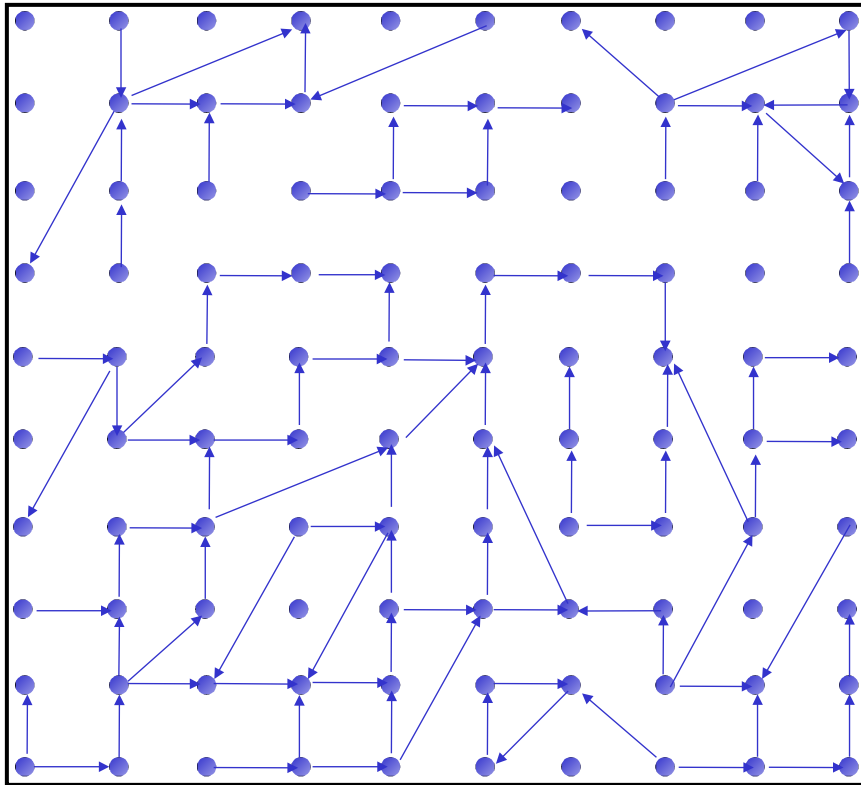


Example

```
Example ( ) {  
1: do{  
    lock();  
    old = new;  
    q = q->next;  
2:   if (q != NULL){  
3:     q->data = new;  
     unlock();  
     new ++;  
    }  
4: } while(new != old);  
5: unlock ();  
   return;  
}
```



What a program *really* is...



State



Transition



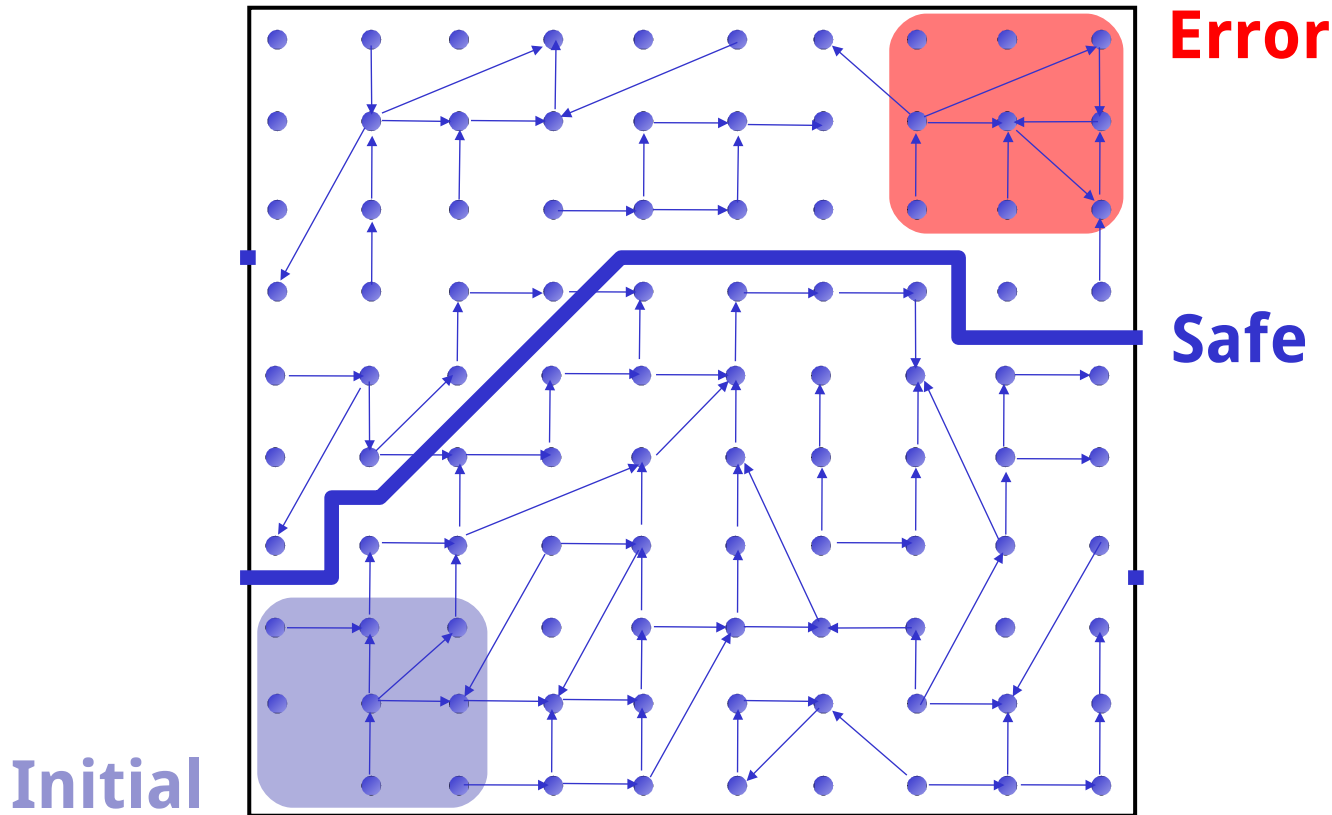
pc → 3
lock → ●
old → 5
new → 5
q → 0x133a

```
3: unlock();  
   new++;  
4: } ...
```

pc → 4
lock → ○
old → 5
new → 6
q → 0x133a

```
Example ( ) {  
1: do{  
   lock();  
   old = new;  
   q = q->next;  
2:   if (q != NULL){  
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   return;}
```

The Safety Verification Problem

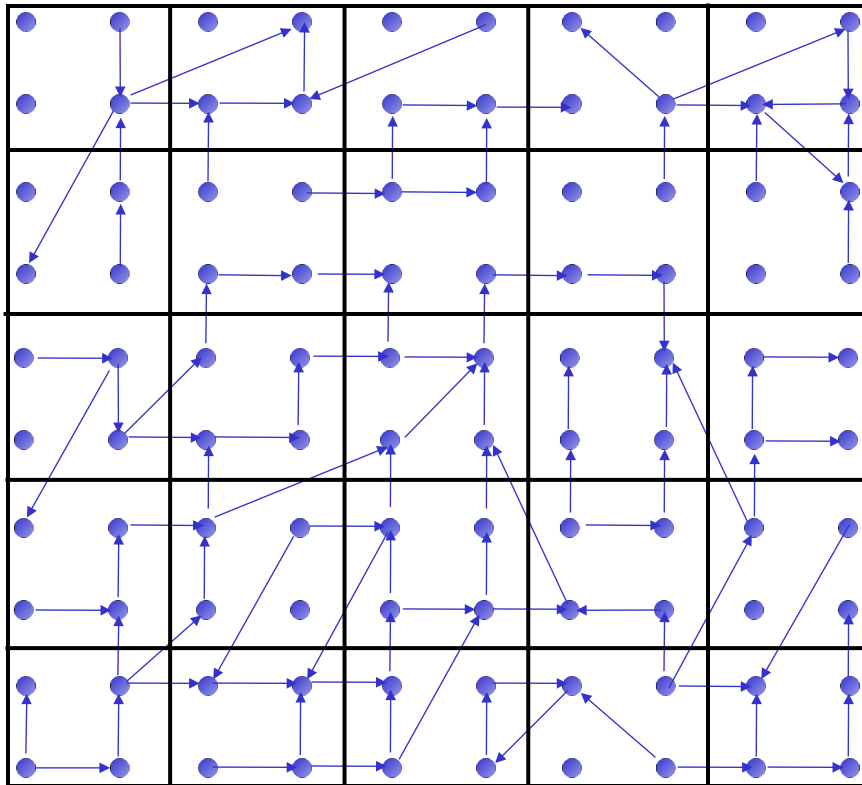


Is there a path from an initial to an error state ?

Problem: Infinite state graph

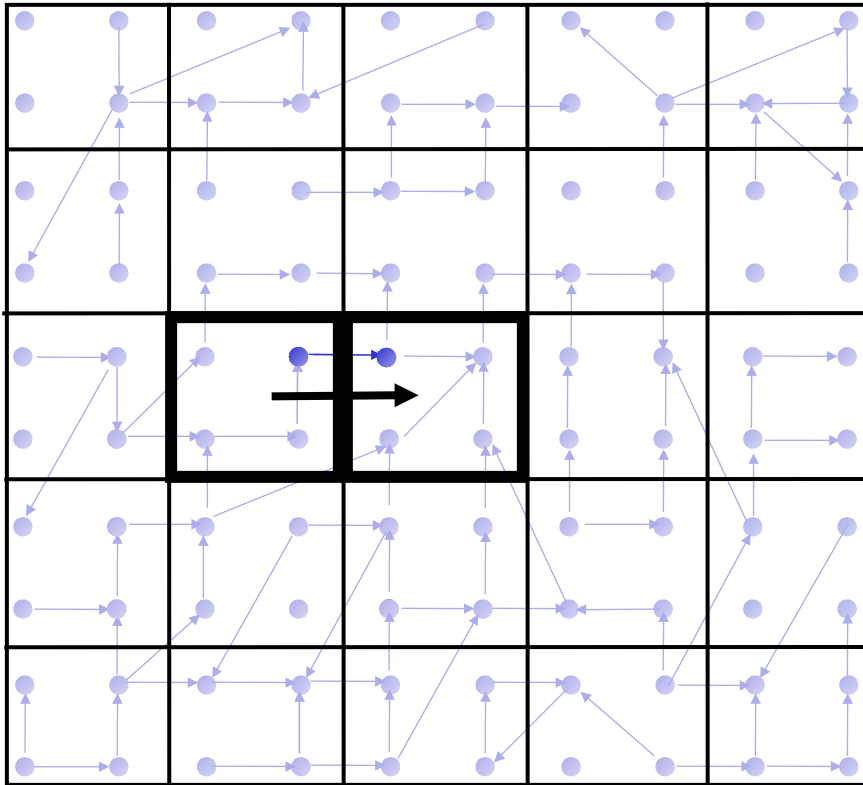
Solution : Set of states ' logical formula

Idea 1: Predicate Abstraction



- Predicates on program state:
lock
old = new
- States satisfying same predicates are equivalent
 - Merged into one abstract state
- #abstract states is finite

Abstract States and Transitions



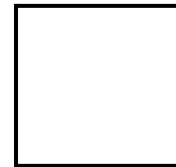
State



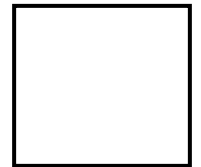
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lock → ●
old → 5
new → 5
q → 0x133a

```
3: unlock();  
   new++;  
4: } ...
```

pc → 4
lock → ○
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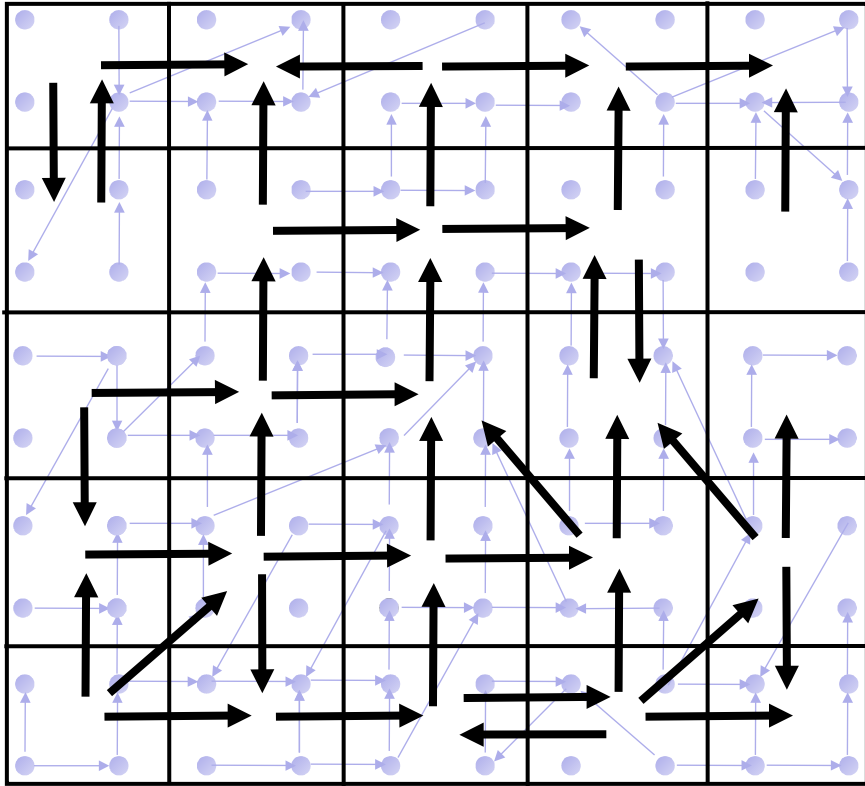


lock
old=new



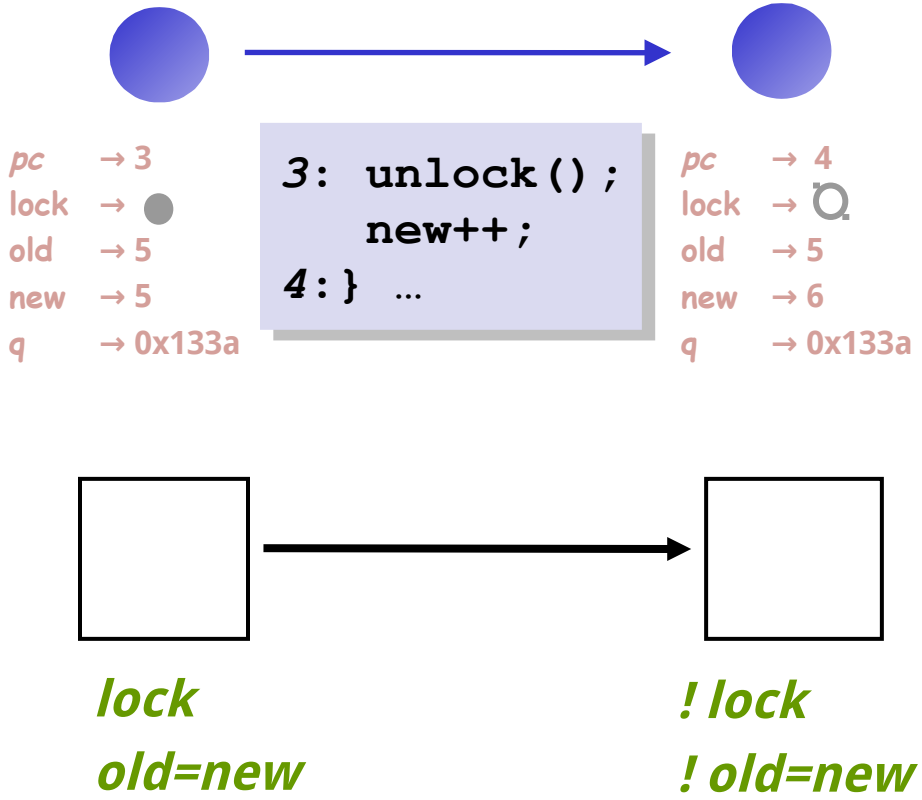
!lock
!old=new

Abstraction

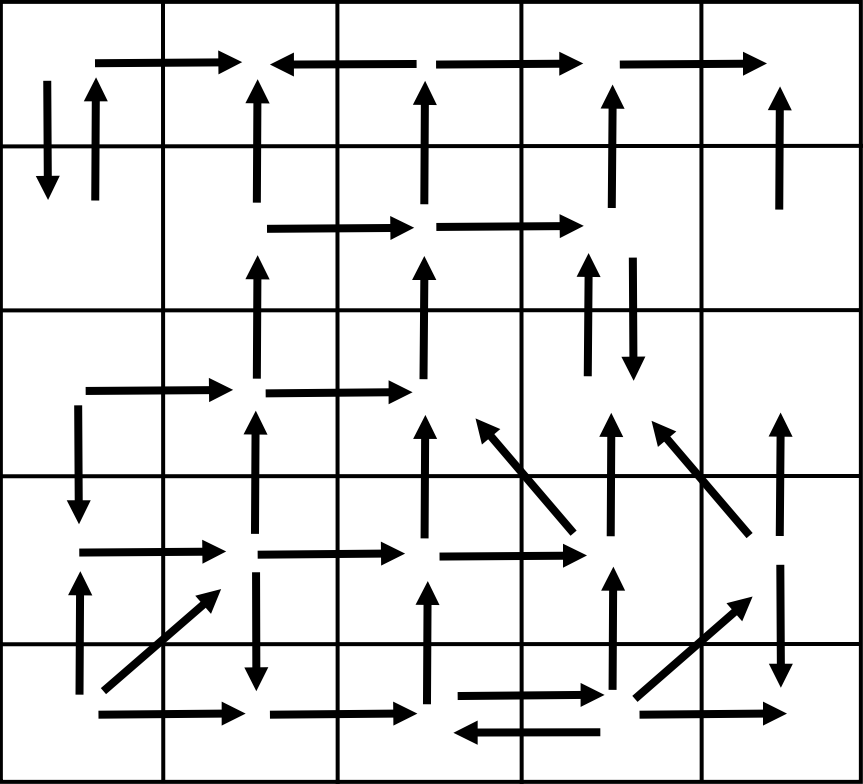


Existential Approximation

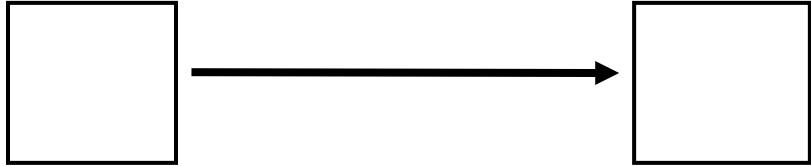
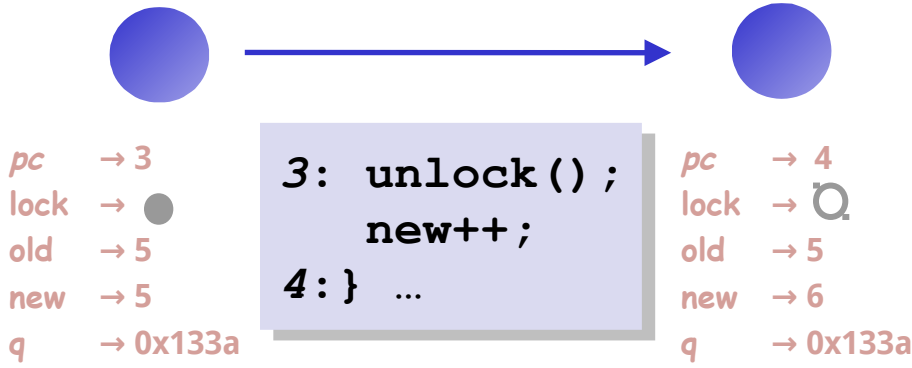
State



Abstraction



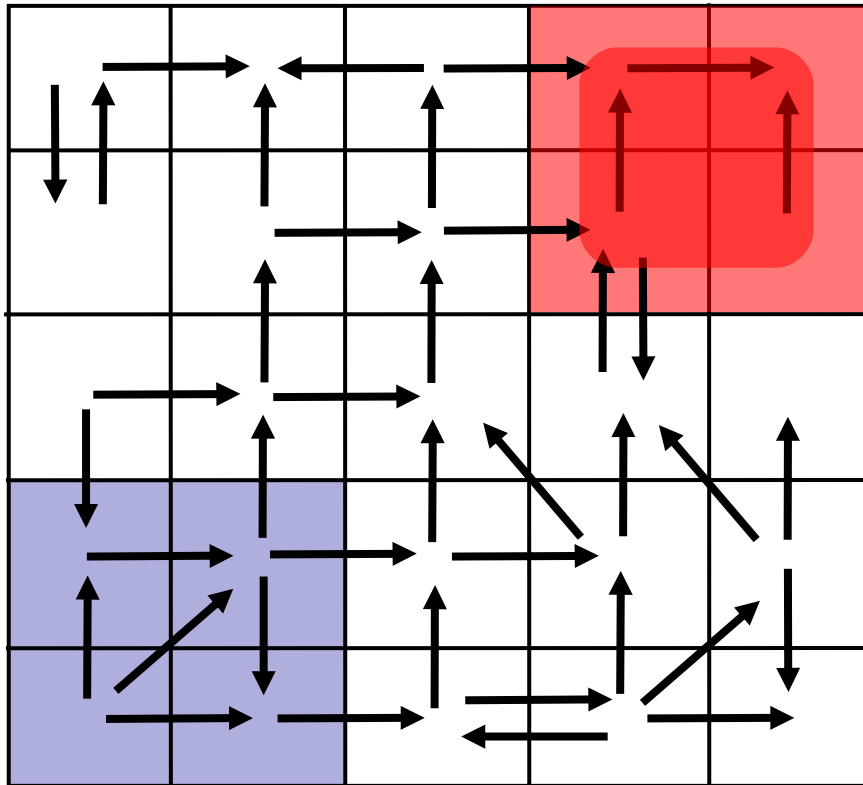
State



lock
old=new

!lock
!old=new

Analyze Abstraction



Analyze finite graph

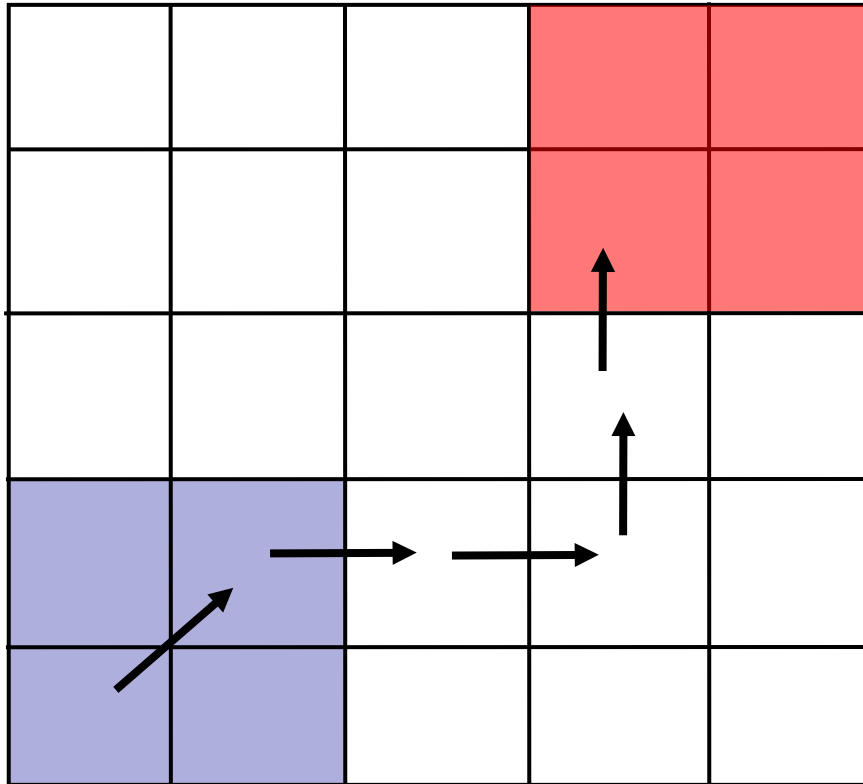
Over Approximate:

Safe => System Safe

Problem

Spurious counterexamples

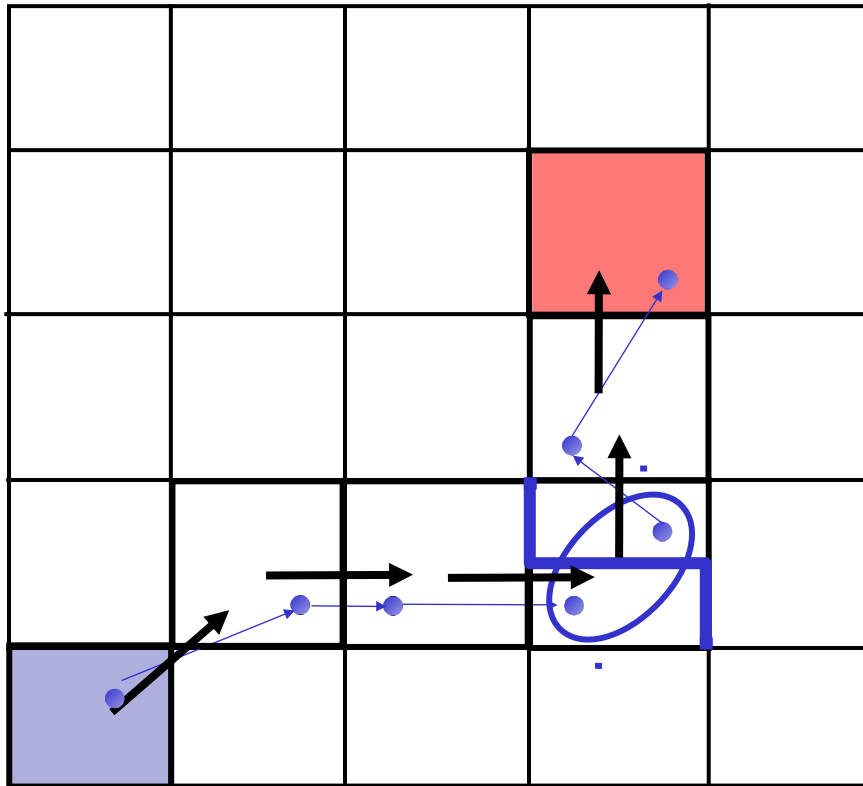
Idea 2: Counterex.-Guided Refinement



Solution

**Use spurious counterexamples
to refine abstraction !**

Idea 2: Counterex.-Guided Refinement

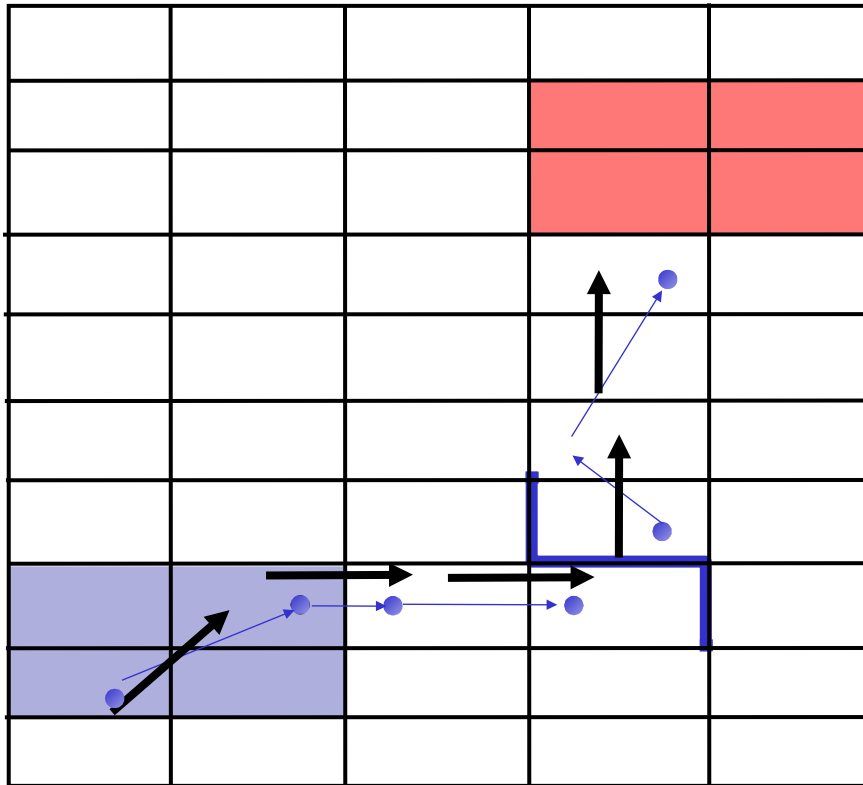


Solution

Use spurious counterexamples to refine abstraction

1. Add predicates to distinguish states across cut
2. Build refined abstraction

Iterative Abstraction-Refinement



[Kurshan et al 93] [Clarke et al 00]
[Ball-Rajamani 01]

Solution

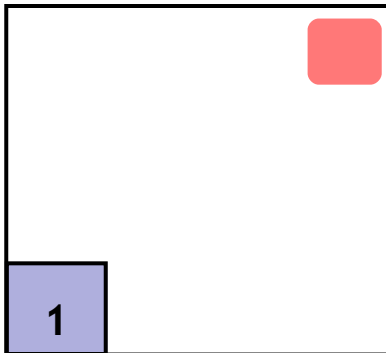
Use **spurious counterexamples**
to **refine abstraction**

1. Add predicates to distinguish states across **cut**
2. Build **refined abstraction**
-eliminates counterexample
3. **Repeat search**
Until real counterexample
or system proved safe

Build-and-Search

```
Example ( ) {  
1: do{  
    lock();  
    old = new;  
    q = q->next;  
2:   if (q != NULL){  
3:     q->data = new;  
       unlock();  
       new ++;  
    }  
4: }while(new != old);  
5: unlock ();  
}
```

1 ! LOCK

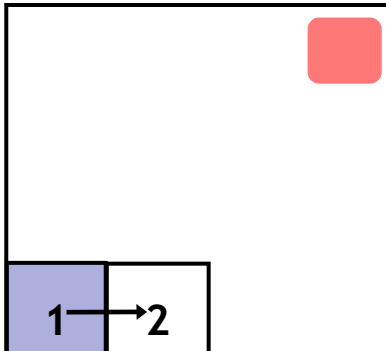
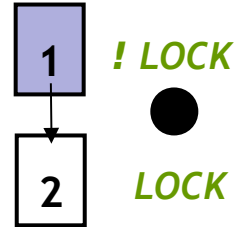


Predicates: *LOCK*

Build-and-Search

```
Example ( ) {  
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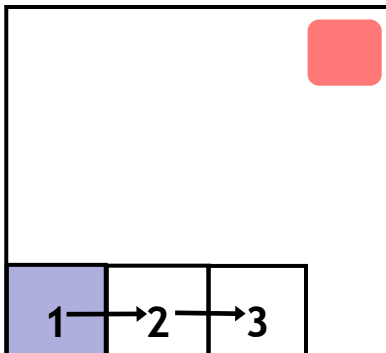
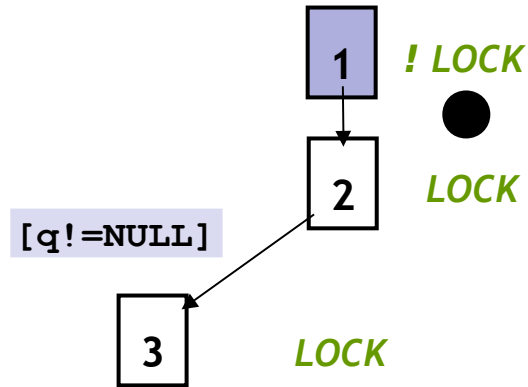
```
lock()  
old = new  
q=q->next
```



Predicates: *LOCK*

Build-and-Search

```
Example ( ) {  
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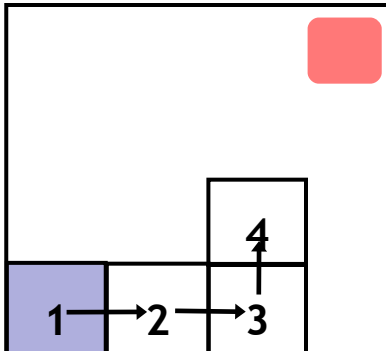
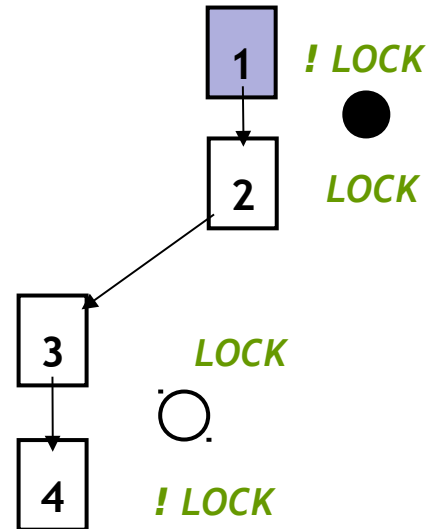


Predicates: **LOCK**

Build-and-Search

```
Example ( ) {  
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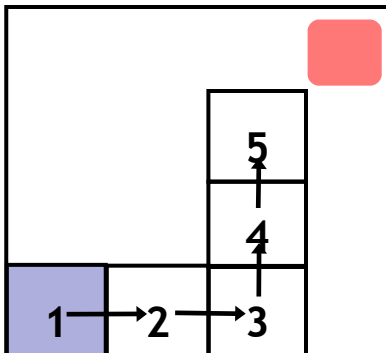
q->data = new
unlock()
new++



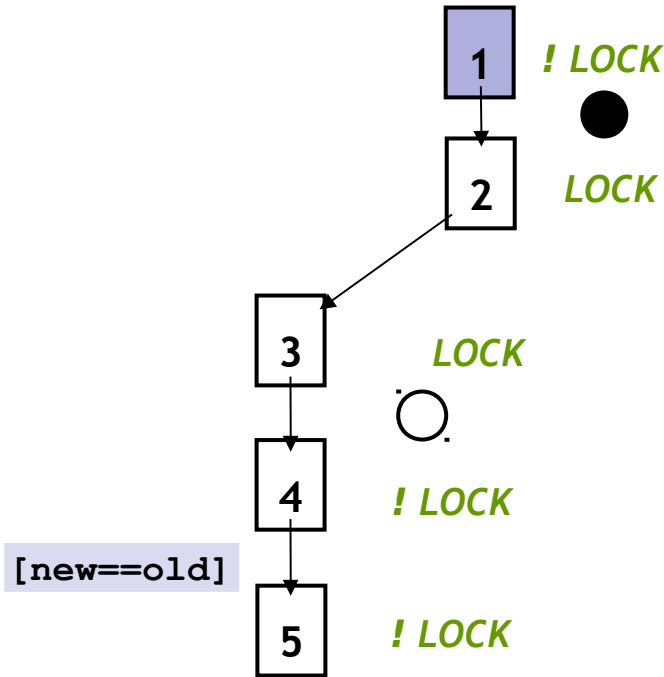
Predicates: *LOCK*

Build-and-Search

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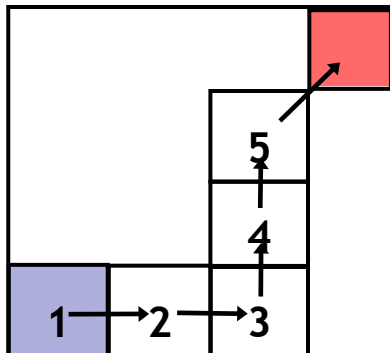


Predicates: *LOCK*

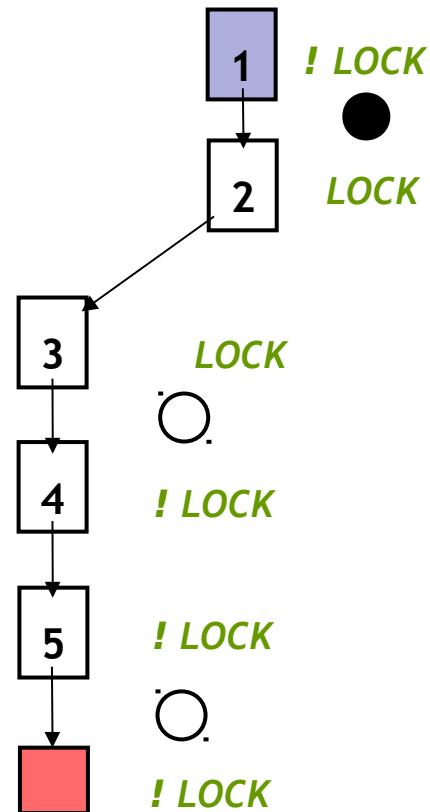


Build-and-Search

```
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Predicates: *LOCK*

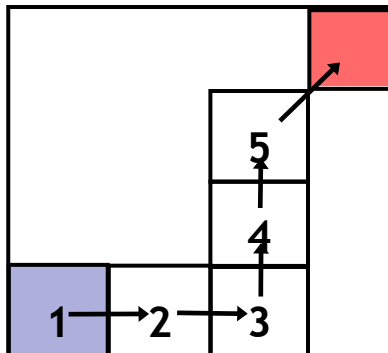


Analyze Counterexample

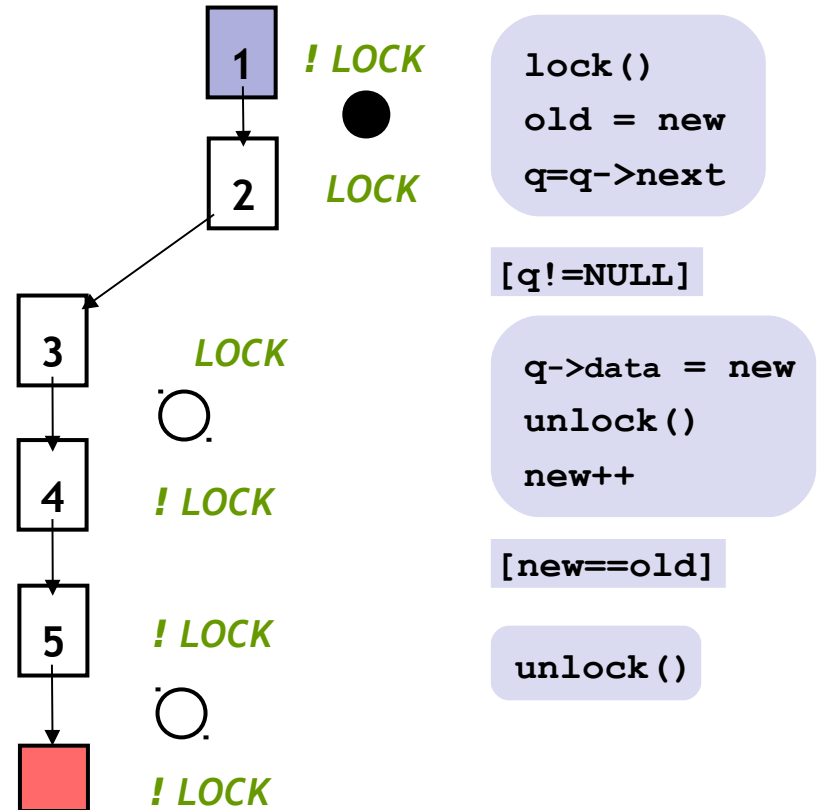
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```



Predicates: *LOCK*

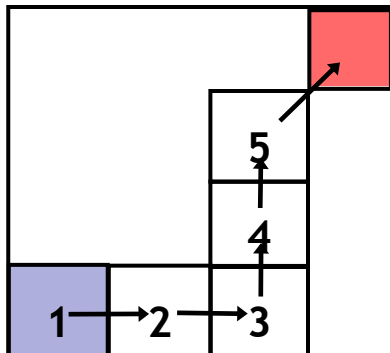


Analyze Counterexample

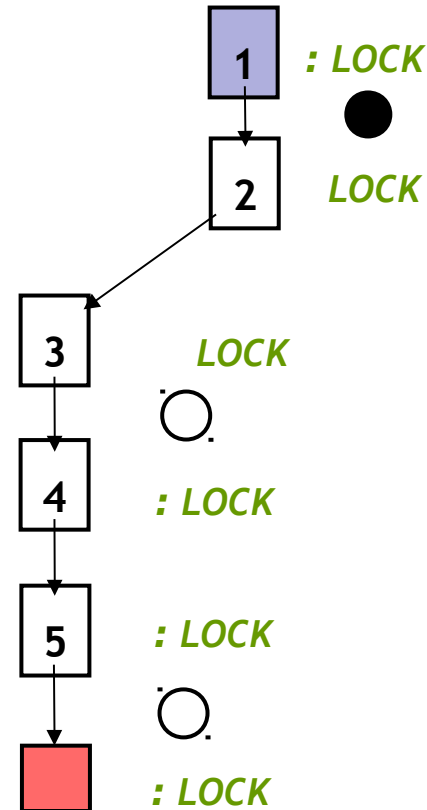
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    unlock();
    new ++;
  }
4: }while(new != old);
5: unlock ();
}

```



Predicates: *LOCK*



old = new

new++

[new==old]

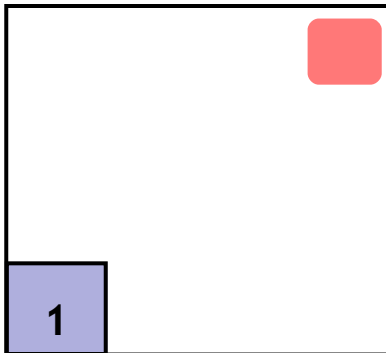
Inconsistent

new == old

Repeat Build-and-Search

```
Example ( ) {  
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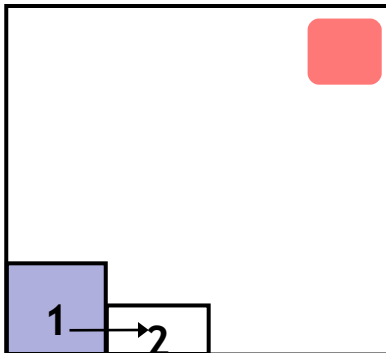
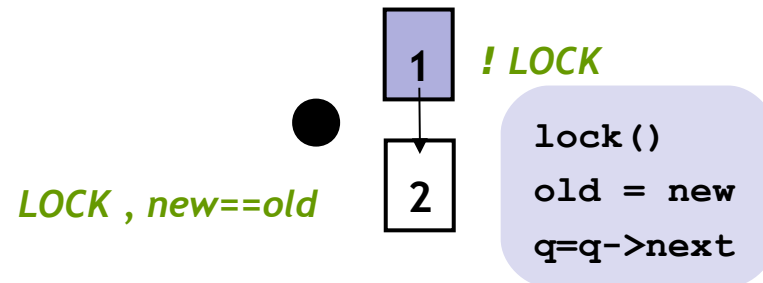
1 : LOCK



Predicates: *LOCK*, *new==old*

Repeat Build-and-Search

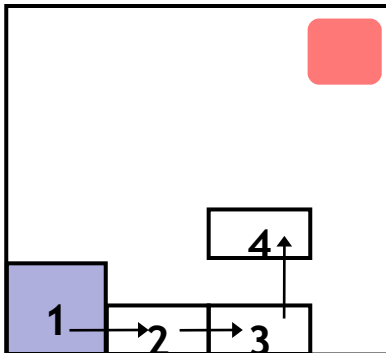
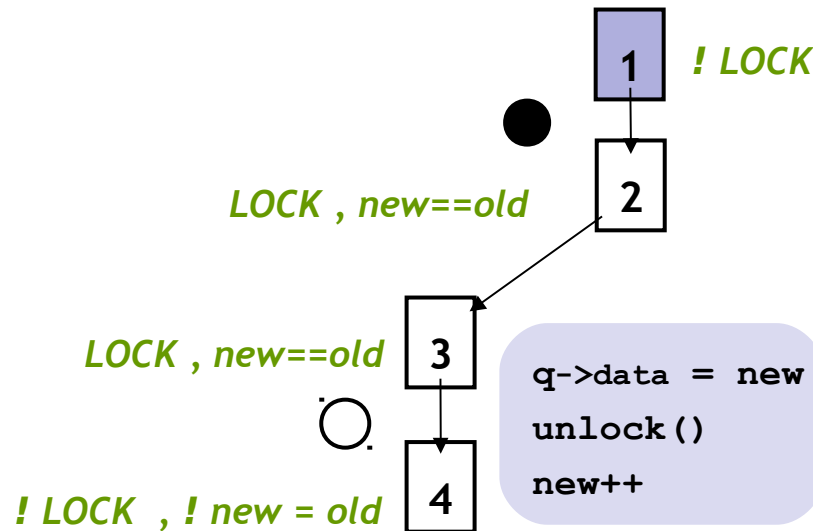
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Predicates: *LOCK, new==old*

Repeat Build-and-Search

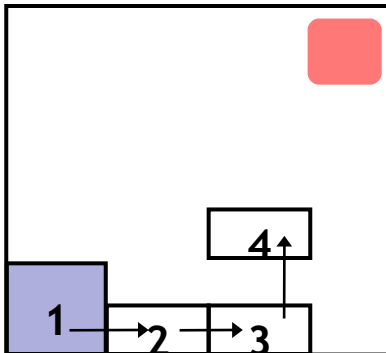
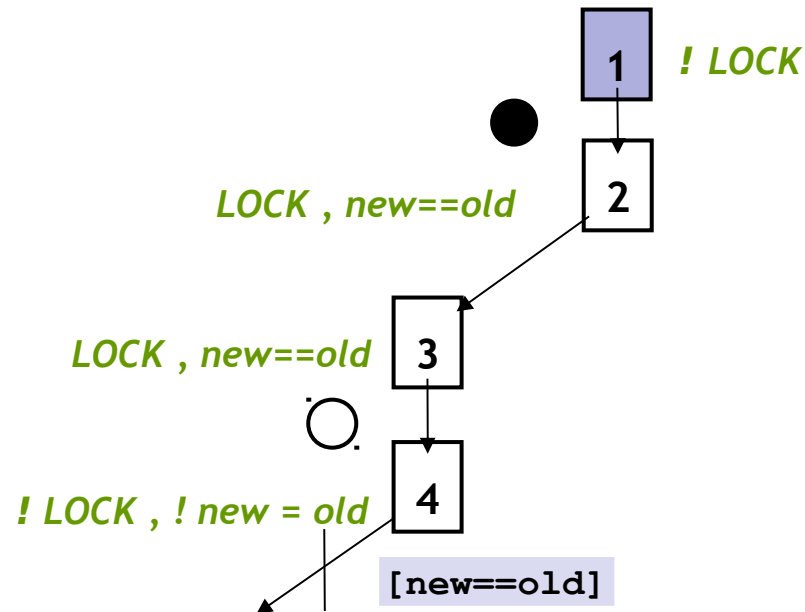
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```



Predicates: **LOCK, new==old**

Repeat Build-and-Search

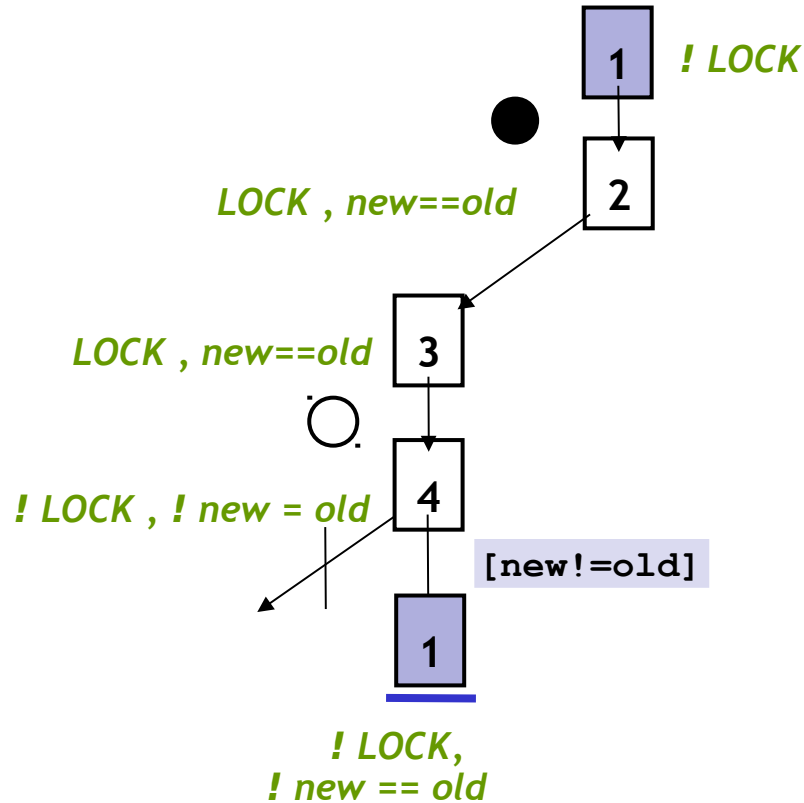
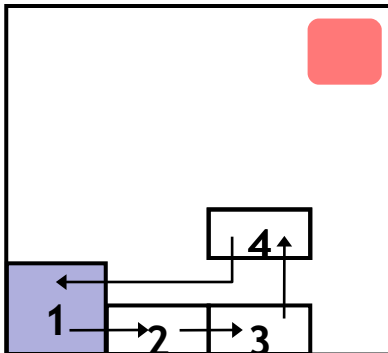
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Predicates: *LOCK*, *new==old*

Repeat Build-and-Search

```
Example ( ) {  
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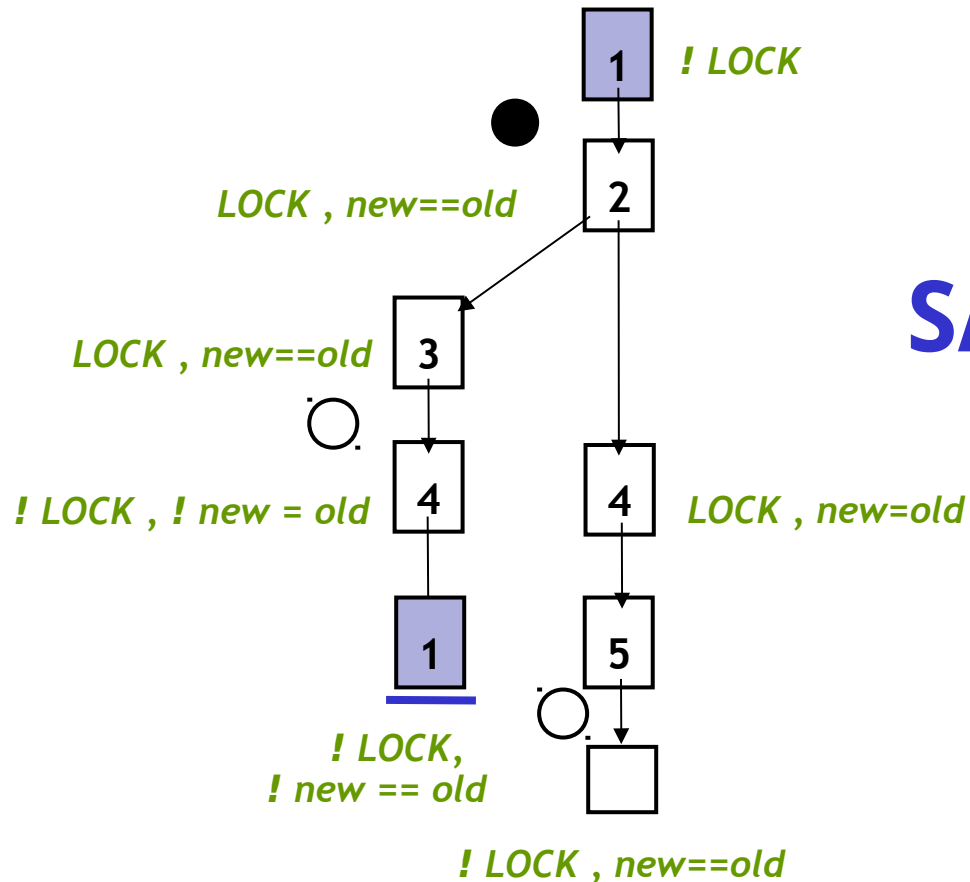
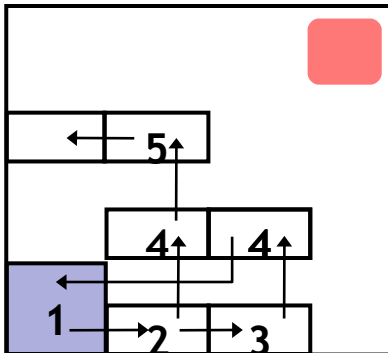
Predicates: **LOCK, new==old**

Repeat Build-and-Search

```

Example ( ) {
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    q = q->next;
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3:     q->data = new;
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    new ++;
  }
4: }while(new != old);
5: unlock ();
}

```



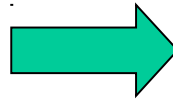
Predicates: *LOCK, new==old*

Another Example

```
1: x = ctr;  
2: y = ctr + 1;  
3: if (x == i-1) {  
4:   if (y != i) {  
      ERROR:  
   }  
}
```

C program

Abstract



```
1: skip;  
2: skip;  
3: if (*) {  
4:   if (*) {  
      ERROR:  
   }  
}
```

**No predicates
available currently**

Checking the abstract model

Is ERROR
reachable?

```
1: skip;  
2: skip;  
3: if (*) {  
4:   if (*) {  
      ERROR:  
   }  
}
```

yes

Abstract
model has a
path leading
to error state

Does this correspond to a real bug?

```
1: skip;  
2: skip;  
3: if (*) {  
4:   if (*) {  
      ERROR:  
   }  
}
```

```
1: x = ctr;  
2: y = ctr + 1;  
3: assume (x == i-1)  
4: assume (y != i)
```

Check using a SAT solver

Not possible

Concrete trace

Refinement

```
1: x = ctr;  
2: y = ctr + 1;  
3: assume (x == i-1)  
4: assume (y != i)
```

Spurious Counterexample

```
1: skip;  
2: skip;  
3: if (*) {  
4:   if (*) {  
      ERROR:  
   }  
}
```

Initial abstraction

Refinement

```
1: x = ctr;  
2: y = ctr + 1;  
3: assume (x == i-1)  
4: assume (y != i)
```

```
1: skip;  
2: skip;  
3: if (*) {  
4:   if (b0) {  
      ERROR:  
   }  
}
```

boolean b0 : y != i

Refinement

```
1: x = ctr;  
2: y = ctr + 1;  
3: assume (x == i-1)  
4: assume (y != i)
```

```
1: skip;  
2: skip;  
3: if (b1) {  
4:   if (b0) {  
      ERROR:  
   }  
}
```

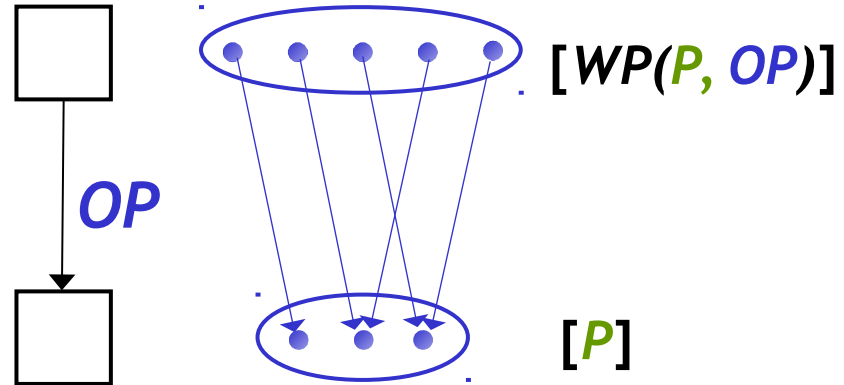
boolean b0 : y != i

boolean b1 : x == i-1

Weakest Preconditions

$WP(P, OP)$

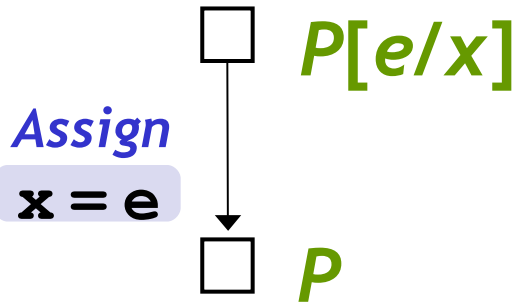
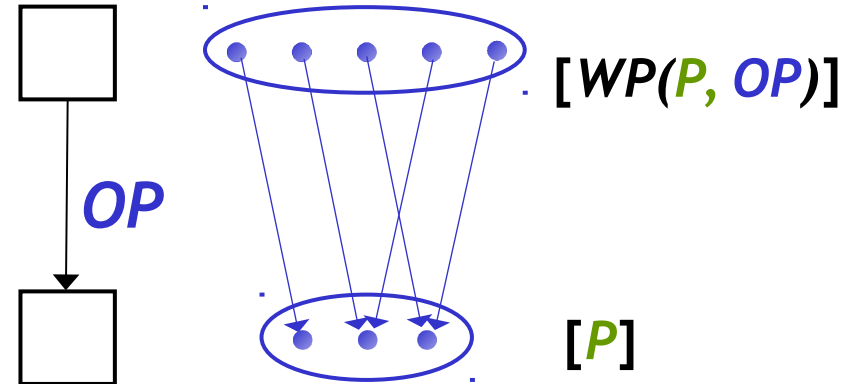
Weakest formula P' s.t.
if P' is true before OP
then P is true after OP



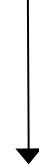
Weakest Preconditions

$WP(P, OP)$

Weakest formula P' s.t.
 if P' is true before OP
 then P is true after OP



$new+1 = old$



$new = old$

$new = new+1$

Refinement

Weakest precondition
of $y \neq i$

$ctr + 1 \neq i$

```
1: x = ctr;  
2: y = ctr + 1;  
3: assume (x == i-1)  
4: assume (y != i)
```

```
1: skip;  
2: b0 = b2;  
3: if (b1) {  
4:   if (b0) {  
        ERROR:  
       }  
   }  
}
```

boolean b0 : $y \neq i$

boolean b1 : $x == i-1$

Refinement

```
1: x = ctr;  
2: y = ctr + 1;  
3: assume (x == i-1)  
4: assume (y != i)
```

boolean b2 : ctr + 1 != i

boolean b3: ctr == i -1

```
1: b1 = b3;  
2: b0 = b2;  
3: if (b1) {  
4:   if (b0) {  
      ERROR:  
   }  
}
```

boolean b0 : y != i

boolean b1 : x == i-1

Refinement

What about
initial values
of b2 and b3?

are
mutually
exclusive.

b2 = 1, b3 = 0
b2 = 0, b3 = 1

So system is
safe!

boolean b2 : ctr + 1 != i

boolean b3: ctr == i - 1

```
1: b1 = b3;  
2: b0 = b2;  
3: if (b1) {  
4:   if (b0) {  
      ERROR:  
   }  
}
```

boolean b0 : y != i

boolean b1 : x == i - 1

Tools for Predicate Abstraction of C

- **SLAM at Microsoft**
 - Used for verifying correct sequencing of function calls in windows device drivers
- **MAGIC at CMU**
 - Allows verification of concurrent C programs
 - Found bugs in MicroC OS
- **BLAST at Berkeley**
 - Lazy abstraction, interpolation
- **SATABS at CMU**
 - Computes predicate abstraction using SAT
 - Can handle pointer arithmetic, bit-vectors
- **F-Soft at NEC Labs**
 - Localization, register sharing